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Exercise 8 for the lecture

Numerics II

WS 2015/2016

http://numerik.mi.fu-berlin.de/wiki/WS_2015/NumericsII.php

Due: Thu, 2016-01-21

Problem 1

Prove the following statements:

a) Let $C \in \mathbb{R}^{n\times n}$ be s.p.d.. Then $A \in \mathbb{R}^{n\times n}$ is symmetric with respect to $\langle \cdot, \cdot \rangle_C$ if and only if CA is symmetric.

b) Let $A \in \mathbb{R}^{n\times n}$ be diagonalizable, then there is an s.p.d. $C \in \mathbb{R}^{n\times n}$ such that A is symmetric with respect to $\langle \cdot, \cdot \rangle_C$.

Problem 2

Let $A \in \mathbb{R}^{n,n}$ be symmetric positive definite and $b \in \mathbb{R}^n$.

a) Show that there is a strictly convex functional $J : \mathbb{R}^n \to \mathbb{R}$ such that the solution of the linear system $Ax = b$ is the minimizer of J.

b) Show that the Richardson iteration

$$x_{k+1} = x_k + \omega(b - Ax_k) \tag{1}$$

with $\omega \in \mathbb{R}$ is equivalent to the explicit Euler method for the gradient flow associated with J.

c) Show that there is an $\omega > 0$ such that the Richardson iteration converges to the solution of the linear system without using the results of Chapter 4 on numerical linear algebra.

d) Why is using an implicit Runge-Kutta method for the gradient flow not a good idea to solve the linear system ?

Please turn over.

Problem 3

Consider the matrix A of the model problem given in the lecture. Compute the convergence rate of the linear iteration obtained if only the block-diagonal part is used as preconditioner.