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1. Übung zur Vorlesung

Numerik IV

WiSe 2015/2016

http://numerik.mi.fu-berlin.de/wiki/WS_2015/NumerikIV.php

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Exercise 1

Let $I = [a, b]$ and $f \in L^2(I)$. We consider the solution $u \in H_0^1(I)$ of the weak Laplace equation,

$$\forall v \in H_0^1(I)\colon \quad \int_I u'(x)v'(x)\mathrm{d}x = \int_I f(x)v(x)\mathrm{d}x,$$

and its discrete approximation $u_S \in S$, which fulfills

$$\forall v \in S \cap H_0^1(I)\colon \quad \int_I u_S'(x)v'(x)\mathrm{d}x = \int_I f(x)v(x)\mathrm{d}x,$$

where S denotes the space of piecewise linear functions over an arbitrary non-trivial grid of equidistant points in I.

Show that the piecewise linear interpolation $I_S u$ of u on S is well-defined and that $I_S u = u_S$.

Exercise 2

Let $n \in \mathbb{N}$ and $\Omega \subseteq \mathbb{R}^n$ with smooth boundary $\partial\Omega$. For $f \in L^2(\Omega)$ we consider the Bi-Laplace equation

$$\Delta^2 u = f \text{ on } \Omega$$

where $\Delta^2 := \Delta\Delta$ is the Bi-Laplacian.

Derive the weak formulation of this problem for $f_0, f_1 \in C^\infty(\overline{\Omega})$ and the boundary conditions

a)

$$u = f_0 \text{ on } \partial\Omega$$
$$\frac{\partial}{\partial n} u = f_1 \text{ on } \partial\Omega.$$

b)

$$\Delta u = f_0 \text{ on } \partial\Omega$$
$$-\frac{\partial}{\partial n} \Delta u = f_1 \text{ on } \partial\Omega.$$