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4th Exercise for the Lecture

NUMERICS IV

WiSe 2015/2016

http://numerik.mi.fu-berlin.de/wiki/WS_2015/NumerikIV.php

Discussion on: Tue, November 24th, 2015

Exercise 1 (mean curvature for signed distance representation)

Let Γ an oriented C^2 -hypersurface with distance function $d: \mathbb{R}^{n+1} \rightarrow \mathbb{R}$. Prove

$$H(x) = \Delta d(x)$$

for all $x \in \Gamma$.

Exercise 2 (mean curvature for graph representation)

Let $\Omega \subseteq \mathbb{R}^n$, $v \in C^2(\Omega)$ and $\Gamma = \{(x, v(x)) \mid x \in \Omega\}$ an orientable C^2 -hypersurface. Furthermore, choose the normal induced by

$$\begin{aligned} \nu_\Omega: \Omega &\longrightarrow \mathbb{R}^{n+1} \\ x &\longmapsto \frac{1}{\sqrt{1 + |\nabla v(x)|^2}} \begin{pmatrix} \nabla v(x) \\ -1 \end{pmatrix}. \end{aligned}$$

Prove

$$H((x, v(x))) = \nabla \cdot \left(\frac{\nabla v(x)}{\sqrt{1 + |\nabla v(x)|^2}} \right)$$

for all $x \in \Omega$.

Exercise 3 (example: mean curvature of a curve)

Let $a, b \in \mathbb{R}_{>0}$ and $\Gamma = \{(a \cos(t), b \sin(t)) \mid t \in [0, 2\pi]\}$. Calculate the mean curvature of Γ .

Have fun!