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12th Exercise for the Lecture

Numerics IV

WiSe 2015/2016

http://numerik.mi.fu-berlin.de/wiki/WS_2015/NumerikIV.php

Discussion on: Tue, February 9th, 2016

Exercise 1 (verifying an equation's viscosity solution)
Let $\Omega := (-1, 1)$. Show that $u\colon \Omega \to \mathbb{R}$, $x \mapsto |x| - 1$ is a viscosity solution of

$$-|u'(x)| + 1 = 0 \text{ on } \Omega, \qquad u(-1) = 0, \qquad u(1) = 0$$

but not of

$$|u'(x)| - 1 = 0 \text{ on } \Omega, \qquad u(-1) = 0, \qquad u(1) = 0.$$

Exercise 2 (uniqueness of a viscosity solution)
Let $\Omega = (-1, 1)$. Then $u\colon \Omega \to \mathbb{R}$, $x \mapsto -|x| + 1$ is the viscosity solution of

$$|u'(x)| - 1 = 0 \text{ on } \Omega, \qquad u(-1) = 0, \qquad u(1) = 0.$$

Show that u is the unique solution of this problem.

Remark: You do not have to prove that u is a viscosity solution of this problem.

Exercise 3 (changing a viscosity solution's sign)
Let $\Omega \subseteq \mathbb{R}^n$ and consider a viscosity solution u of a second order PDE of the type

$$F(x, u, Du, D^2u) = 0,\ x \in \Omega.$$

Prove that $-u$ is then a viscosity solution of

$$-F(x, -u, -Du, -D^2u) = 0,\ x \in \Omega.$$

Have fun!