

Exercise 1 for the lecture

NUMERICS II

WS 2017/2018

http://numerik.mi.fu-berlin.de/wiki/WS_2017/NumericsII.php

Due: Wed, 31-10-2017

Problem 1 (4 TP)

Let

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & -1 \end{pmatrix}.$$

- a) Write A as a sum of a diagonalizable matrix D and nilpotent matrix N .
- b) Calculate the matrix exponential e^{tA} .

Problem 2 (6 TP)

Show that the matrix exponential has the following properties

a)

$$e^{t(A+B)} = e^{tA}e^{tB}, \quad \forall A, B \in \mathbb{R}^{d \times d} \text{ with } AB = BA$$

b)

$$A = \text{blockdiag}(A_1, \dots, A_k) \Rightarrow e^{tA} = \text{blockdiag}(e^{tA_1}, \dots, e^{tA_k})$$

c)

$$e^{\alpha I} = e^\alpha I, \quad \alpha \in \mathbb{R}, I = \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix} \in \mathbb{R}^{d \times d}.$$

Problem 3 (4 TP)

Consider the following system of linear differential equations

$$x'(t) = Ax(t), \quad t > 0 \tag{1}$$

with the symmetric matrix

$$A = \begin{pmatrix} \frac{-1591}{25} & 0 & \frac{1212}{25} \\ 0 & -1 & 0 \\ \frac{1212}{25} & 0 & \frac{-884}{25} \end{pmatrix}.$$

- a) Calculate all solutions of (1).
- b) Give the solutions for the initial values $x_1(t_0) = (\frac{3}{5}, 0, \frac{4}{5})$ and $x_2(t_0) = (-\frac{4}{5}, 0, \frac{3}{5})$ and sketch their trajectories.

GENERAL REMARKS

You have to do the exercises in groups of up 3 people. Be prepared to demonstrate your solutions to theoretical problems at the given date in the tutorial. Solutions for programming problems have to be submitted via e-mail to adjurdjevac@mi.fu-berlin.de with a subject starting by [NumericsII] and denoting all members of the group. Please follow the additional advise for programming exercises on the homepage.