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Exercise 3 for the lecture

# Numerics II

WS 2017/2018

http://numerik.mi.fu-berlin.de/wiki/WS_2017/NumericsII.php

**Due: Wed, 8-11-2017**

**Problem 1** (6 TP)
Let $A \in \mathbb{R}^{d\times d}$ and $x^* = 0$ is a fixed point. Prove the following statements.

a) If $\nu(A) = 0$, then $x^*$ is not stable.

b) If $\nu(A) \leq 0$ and there exists a $\lambda \in \sigma(A)$ such that $\mathcal{R}(\lambda) = 0$ but $r(\lambda) < s(\lambda)$, then $x^*$ is not stable.

c) If $\nu(A) \leq 0$ and there exists a $\lambda \in \sigma(A)$ such that $\mathcal{R}(\lambda) = 0$ and $r(\lambda) = s(\lambda)$, then $x^*$ is not asymptotically stable.

**Problem 2** (4 TP)
Let function $f$ be Lipschitz continuous with Lipschitz constant $L$ and $f(0) = 0$. Furthermore, we assume that $x \in \mathbb{R}^d$ solves the initial value problem

$$x'(t) = f(x(t)), \quad x(0) = x_0$$

for $t \in [0, T]$. Prove the following bound

$$\|\Phi^t x_0\| \leq \|x_0\| e^{Lt},$$

where $\Phi^t$ is the flow operator.

(Hint: Use Gronwall's Lemma.)

**Problem 3** (5 TP)
Consider the following system of ODEs

$$x'(t) = f(x(t)), \quad f = \begin{bmatrix} x_2 \\ \mu(1 - x_1^2)x_2 - x_1 \end{bmatrix} \tag{1}$$

where $\mu$ is a real parameter.

a) Calculate all fixed points of (1).

b) Discuss the (asymptotic) stability of these fixed points depending on parameter $\mu$.

## General remarks

You have to do the exercises in groups of up 3 people. Be prepared to demonstrate your solutions to theoretical problems at the given date in the tutorial. Solutions for programming problems have to be submitted via e-mail to `adjurdjevac@mi.fu-berlin.de` with a subject starting by [NumericsII] and denoting all members of the group. Please follow the additional advise for programming exercises on the homepage.