Fachbereich Mathematik & Informatik
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Exercise 5 for the lecture

Numerics II

WS 2017/2018

http://numerik.mi.fu-berlin.de/wiki/WS_2017/NumericsII.php

Due: Tue, 21-11-2017

Problem 1 (3 TP)
Let $\Phi^t = \exp(\lambda t)$. Show that if Ψ^τ is consistent with Φ^t with order p, then

$$\Psi^\tau = R(z) = \exp(z) + \mathcal{O}(z^{p+1}) \qquad \text{for } z \to 0$$

with $z = \lambda\tau$.

Problem 2 (5 TP)

a) Find polynomials P and Q with degree two that satify

$$\exp(z) = \frac{P(z)}{Q(z)} + \mathcal{O}(z^5) \quad \text{for } z \to 0.$$

b) Using the command `contour` in Matlab, plot the stability domain of the Runge-Kutta method with the stability function

$$R(z) = \frac{P(z)}{Q(z)}$$

where P and Q are the polynomilas from part (a). Prove the A-stability of this method. Is the method L-stable?

c) Implement the scheme corresponding to $R(z)$ and test it on the scalar problem

$$x'(t) = 2x(t), \quad x(0) = 1$$

and on the twodimensional problem

$$x'(t) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} x(t), \quad x(0) = (11)^T.$$

In both cases plot the numerical as well as the exact solution.

Problem 3 (4 TP)

a) Compute the time step restriction for the Runge-Kutta-4 method applied to the linear system

$$x'(t) = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} x(t). \tag{1}$$

b) Sketch the stability domain for the method of Runge and visualize the time step restriction for (1).

Problem 4 (4 TP)

a) Can the equation $(x^2 + y^3 + 2z^2)^{\frac{1}{2}} = \cos z$ be solved uniquely for y in terms of x and z near $(0, 1, 0)$? What about z in terms of x and y?

b) Suppose $F(x, y) \in C^1$ and $F(0, 0) = 0$. What conditions on F will guarantee that $F(F(x, y), y) = 0$ can be solved for y as a C^1 function of x near $(0, 0)$?

c) Let (x, y) denote a point in Cartesian coordinates and let (r, θ) denote a point in polar coordinates

$$x = r\cos\theta, \quad y = r\sin\theta.$$

For which values of x and y can we solve these two equaions for r and θ in terms of x and y?

General remarks

You have to do the exercises in groups of up 3 people. Be prepared to demonstrate your solutions to theoretical problems at the given date in the tutorial. Solutions for programming problems have to be submitted via e-mail to `adjurdjevac@mi.fu-berlin.de` with a subject starting by [NumericsII] and denoting all members of the group. Please follow the additional advise for programming exercises on the homepage.