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Exercise 12 for the lecture

Numerics II

WS 2017/2018

http://numerik.mi.fu-berlin.de/wiki/WS_2017/NumericsII.php

Due: Wed, 24-01-2018

Problem 1 (4 TP)

a) Show that the parallel directional correction method associated with the Euclidean unit vectors e_i is the Jacobi method.

b) Show that the successive directional correction method associated with the Euclidean unit vectors e_i is the Gauß-Seidel method.

Problem 2 (4 TP)
Let A be a symmetric positive definite matrix and let B be a matrix that satisfies

$$\langle Ax, x\rangle < 2\langle Bx, x\rangle \qquad \forall x, y.$$

a) Show that $B + B^T - A$ is an s.p.d. matrix.

b) Show that the method

$$x^{k+1} = x^k + B^{-1}(b - Ax^k)$$

converges.

c) Show that the Gauss-Seidel method converges for any x^0, if the matrix A is s.p.d..

Hint: In order to prove part b), consider the matrix $M := -B^{-1}(A - B)$ and show that $\|My\|_A < 1$ or $\rho(M) < 1$.

Problem 3 (2 TP + 3 PP)
Consider the the linear system

$$AU = b \tag{1}$$

with the symmetric positive definite matrix $A \in \mathbb{R}^{n,n}$ and $b \in \mathbb{R}^n$.

a) Compute an upper bound for the convergence rate of the Jacobi method applied to the linear system (1) with the matrix A obtained by a finite difference discretization of the Poisson equation using a uniform grid on $[0,1] \times [0,1]$ given in the lecture.

b) Implement the Jacobi and the Gauß-Seidel methods in `matlab` as

```
function [u, uk] = Jacobi(A, b, u0, tol, uexact)
```

and

```
function [u, uk] = GaussSeidel(A, b, u0, tol, uexact).
```

`u`, uk, `A`, `b`, `u0`, tol, and u$exact$ denote the last iterate, a vector containing all iterates, the system matrix, the right hand side, the initial iterate, the error tolerance, and the exact solution, respectively. The iteration should stop if the energy norm $\|\cdot\|_A = \langle A\cdot, \cdot\rangle^{0.5}$ of the error is smaller than the tolerance. Test your programm with the matrix of part a) and the right hand side $b = AU$ where U is the point wise evaluation of $(x_1 - x_1^2)(x_2 - x_2^2)$ for `u0` $= 0$, t$ol = 10^{-8}$ and various choices of n. Plot the error over the number of iteration steps and compute the average convergence rate for each choice of n.

General remarks

You have to do the exercises in groups of up 3 people. Be prepared to demonstrate your solutions to theoretical problems at the given date in the tutorial. Solutions for programming problems have to be submitted via e-mail to `adjurdjevac@mi.fu-berlin.de` with a subject starting by [NumericsII] and denoting all members of the group. Please follow the additional advise for programming exercises on the homepage.