Fachbereich Mathematik & Informatik
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Exercise 13 for the lecture

Numerics II

WS 2017/2018

http://numerik.mi.fu-berlin.de/wiki/WS_2017/NumericsII.php

Due: Wed, 31-01-2018

Problem 1 (4 TP)
Show that a convergent linear iteration converges linearly with respect to B norm if the system matrix A and the preconditioner B of the iteration are s.p.d..

Problem 2 (5 TP)
Let A and B be symmetric positive definite matrices. Prove the convergence of the so-called preconditioned gradient method defined by

$$x^{k+1} = x^k + \alpha_k B^{-1}(b - Ax^k)$$

where

$$\alpha_k = \arg\min_{\alpha \in \mathbb{R}} J(x^k + \alpha B^{-1}(b - Ax^k)).$$

Hints:

a) Show that $\|y - x\|_A^2 = 2J(y) + \|x\|_A^2$ for all $y \in \mathbb{R}$.

b) Show that there is a fixed $\omega > 0$, such that the modified method with α_k replaced by ω, reduces the error and the energy.

c) Show that the error $\|x^k - x\|_A$ converges faster than the error from the modified method.

Problem 3 (4 TP - extra points)
Derive from the CG method a method for *non-symmetric* A by application of A^T to $Ax = b$. Which Krylov spaces are spanned by this method? What can you say about the convergence properties?

General remarks

You have to do the exercises in groups of up 3 people. Be prepared to demonstrate your solutions to theoretical problems at the given date in the tutorial. Solutions for programming problems have to be submitted via e-mail to `adjurdjevac@mi.fu-berlin.de` with a subject starting by [NumericsII] and denoting all members of the group. Please follow the additional advise for programming exercises on the homepage.