

Exercise 3 for the lecture

NUMERICS II

WS 2017/2018

http://numerik.mi.fu-berlin.de/wiki/WS_2017/NumericsII.php

Due: Wed, 8-11-2017

Problem 1 (6 TP)

Let $A \in \mathbb{R}^{d \times d}$ and $x^* = 0$ is a fixed point. Prove the following statements.

- a) If $\nu(A) > 0$, then x^* is not stable.
- b) If $\nu(A) \leq 0$ and there exists a $\lambda \in \sigma(A)$ such that $\mathcal{R}(\lambda) = 0$ but $r(\lambda) < s(\lambda)$, then x^* is not stable.
- c) If $\nu(A) \leq 0$ and there exists a $\lambda \in \sigma(A)$ such that $\mathcal{R}(\lambda) = 0$ and $r(\lambda) = s(\lambda)$, then x^* is not asymptotically stable.

Problem 2 (4 TP)

Let function f be Lipschitz continuous with Lipschitz constant L and $f(0) = 0$. Furthermore, we assume that $x \in \mathbb{R}^d$ solves the initial value problem

$$x'(t) = f(x(t)), \quad x(0) = x_0$$

for $t \in [0, T]$. Prove the following bound

$$\|\Phi^t x_0\| \leq \|x_0\| e^{Lt},$$

where Φ^t is the flow operator.

(Hint: Use Gronwall's Lemma.)

Problem 3 (5 TP)

Consider the following system of ODEs

$$x'(t) = f(x(t)), \quad f = \begin{bmatrix} x_2 \\ \mu(1 - x_1^2)x_2 - x_1 \end{bmatrix} \quad (1)$$

where μ is a real parameter.

- a) Calculate all fixed points of (1).
- b) Discuss the (asymptotic) stability of these fixed points depending on parameter μ .

GENERAL REMARKS

You have to do the exercises in groups of up to 3 people. Be prepared to demonstrate your solutions to theoretical problems at the given date in the tutorial. Solutions for programming problems have to be submitted via e-mail to adjurdjevac@mi.fu-berlin.de with a subject starting by [NumericsII] and denoting all members of the group. Please follow the additional advice for programming exercises on the homepage.