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Exercise 3 for the lecture

# Numerics II

WS 2019/2020

http://numerik.mi.fu-berlin.de/wiki/WS_2019/NumericsII.php

**Due: Thursday, November 7th at the tutorial**

**1. Exercise** (4 TP)

a) Rewrite the system

$$\begin{aligned} x_1' &= \alpha x_1 - \beta x_2 \\ x_2' &= \beta x_1 + \alpha x_2 \end{aligned}$$

as a scalar complex ode.

b) Derive conditions on $\alpha$, $\beta$ for stability and asymptotic stability of the fixed point $x^* = 0$.

c) Discuss the phase portrait of solutions in the stable, asymptotically stable and unstable case in the phase plane $(x_1, x_2)$.
Explanation (phase portrait): For some purposes, too much information is shown in the classical $(x, t)$- phase plane. If we were solely interested in where solutions curves end up in the limit as $t \to \infty$, we might eliminate the $t$-axis, and simply let arrows indicate where solution trajectories are headed. Loosely speaking, a phase portrait is exactly the figure, which supresses the $t$-axis. The arrows on a phase portrait should indicate the direction of the solution and replace the $t$-axis.

**2. Exercise** (4 TP)
Consider the following system of ODEs

$$x'(t) = f(x(t)), \quad f = \begin{bmatrix} x_2 \\ \mu(1 - x_1^2)x_2 - x_1 \end{bmatrix} \tag{1}$$

where $\mu$ is a real parameter.

a) Calculate all fixed points of (1).

b) Discuss the (asymptotic) stability of these fixed points depending on parameter $\mu$.

**3. Exercise** (4 TP)
Discuss the stability of the fixed point $x^* = 0$ of the following odes

a)

$$\begin{aligned} x_1' &= -x_1 + x_2^2 \\ x_2' &= -\exp(x_1)x_2 \end{aligned}$$

b)

$$\begin{aligned} x_1' &= \cos(x_1) - \exp(-x_2) \\ x_2' &= x_1 x_2 \end{aligned}$$

**4. Exercise** (4 TP)
Prove or disprove: Consider the non-linear system

$$x' = f(x). \tag{2}$$

The linearization of the corresponding flow, at the fixpoint $x^* = 0$ is exactly the flow of the linearized vector field $f$ at $x^* = 0$.

## General remarks

You have to do the exercises in groups of up 3 people. Be prepared to demonstrate your solutions to theoretical problems at the given date in the tutorial. Solutions for programming problems have to be submitted via e-mail to xingjian@zedat.fu-berlin.de. with a subject starting by [NumericsII] and denoting all members of the group. Please follow the additional advise for programming exercises on the homepage.