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Exercise 4 for the lecture

Numerics II

WS 2019/2020

http://numerik.mi.fu-berlin.de/wiki/WS_2019/NumericsII.php

Due: Thursday, November 14th at the tutorial

1. Exercise (3TP)
Assume that the stability of the fixed point $x^* = 0$ of

$$x' = \lambda x \tag{1}$$

implies the stability of the linear recursion

$$x_{k+1} = \Psi^\tau x_k \tag{2}$$

with $\Psi^\tau = R(\lambda\tau)$. Show that

$$\mathbb{C}_- = \{\lambda \in \mathbb{C} \,|\, Re(\lambda) \leq 0\} \subset S = \{z \in \mathbb{C} \,|\, |R(z)| \leq 1\}. \tag{3}$$

2. Exercise (2TP + 2TP)
We consider the system

$$x' = f(x) \tag{4}$$

with $f \in C^1(\mathbb{R}^d)$ with fixed point $x^* = 0$.

a) Show that $\nu(A) > 0$ with $A = Df(x^*)$ implies instability of x^*.

b) Illustrate this general result by an example with $d = 2$.

3. Exercise (3TP)
Consider the growth of bacteria

$$x' = qx - kx^2, \tag{5}$$

with $k > 0$ and $q = 0$. Determine the fixed point x^* of (5) and discuss its (asymptotic) stability from a general and a problem specific perspective.

4. Exercise (1TP + 2TP + 2TP)
We consider the system $x'(t) = Ax(t)$ with the fixed point $x^* = 0$.

a) Let $x^* = 0$ be asymptotically stable. Find additional sufficient conditions on the spectrum $\sigma(A)$ and the stepsize τ for asymptotic stability of the linear recursion

$$x_{k+1} = (I + \tau A)x_k, \quad k = 0, \ldots \tag{6}$$

b) Let all eigenvalues of A be complex (not real) and let $x^* = 0$ be stable and not asymptotically stable. Then the linear recursion is unstable for all $\tau > 0$.

c) Illustrate the result of b) in the special case

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

by computing explicit Euler approximations with a corresponding `Matlab` programm for the initial value $x_\varepsilon = (\varepsilon, \varepsilon)^T$ with $\varepsilon = 10^{-2}, 10^{-4}, 10^{-6}$ and suitable final time T and stepsize $\tau > 0$. What happens, if the implicit Euler method is used?

General remarks

You have to do the exercises in groups of up 3 people. Be prepared to demonstrate your solutions to theoretical problems at the given date in the tutorial. Solutions for programming problems have to be submitted via e-mail to xingjian@zedat.fu-berlin.de. with a subject starting by [NumericsII] and denoting all members of the group. Please follow the additional advise for programming exercises on the homepage.