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Exercise 5 for the lecture

Numerics II

WS 2019/2020

http://numerik.mi.fu-berlin.de/wiki/WS_2019/NumericsII.php

Due: Thursday, November 21th at the tutorial

1. Exercise (2TP)

Let $\Phi^t = \exp(\lambda t)$. Show that if Ψ^τ is consistent with Φ^t with order p, then

$$\Psi^\tau = R(z) = \exp(z) + \mathcal{O}(z^{p+1}) \qquad \text{for } z \to 0$$

with $z = \lambda\tau$.

2. Exercise (2PP + 2PP + 2TP)

a) Implement the (possibly implicit) Runge-Kutta method given by the Butcher scheme b, A for the linear system:

$$x'(t) = Mx(t), \qquad t \in (\mathtt{I}(1), \mathtt{I}(2)] \qquad x(\mathtt{I}(1)) = \mathtt{x}_0$$

in `matlab` as `function` [x, t, k] = RungeKuttaLinear(M, x0, I, tau, b, A), where `M`, `x0`, `I`, and `tau` denote the system matrix, the initial value, the time interval and the step size and `b`, `A` are the entries of the Butcher scheme, respectively. The returned values `x` ($d \times n$ matrix), `t` ($1 \times n$ vector), and `k` should contain the solution at each time step, the time steps, and the intermediate $d \times s$ vectors k_i for all time steps, respectively.

b) Test your program with the initial value problem

$$x'(t) = -\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} x(t), \qquad x(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{1}$$

on the interval $(0, 5]$ with the step sizes $\tau = 10^{-3}, 10^{-2}, 10^{-1}, 1$ for the method of Runge, and the Gauß method of order 4. Plot the discretization error and discuss the numerical results.

c) Evaluate the collocation polynomials u of the Gauß method of order 4 applied to the above problem with step size $\tau = 1$ from the intermediate vectors k_i on a sufficiently fine sample grid and plot the discrete trajectories given by the values of u.

3. Exercise (2TP)
Find an implicit Runge-Kutta method which is implicit and consistent, but not A-stable.

4. Exercise (2TP)
Compute the Butcher scheme for the collocation method with the supporting points of the Simpson rule.

General remarks

You have to do the exercises in groups of up 3 people. Be prepared to demonstrate your solutions to theoretical problems at the given date in the tutorial. Solutions for programming problems have to be submitted via e-mail to xingjian@zedat.fu-berlin.de. with a subject starting by [NumericsII] and denoting all members of the group. Please follow the additional advise for programming exercises on the homepage.