Fachbereich Mathematik & Informatik
Freie Universität Berlin
Prof. Dr. Ralf Kornhuber, Xingjian Zhang-Schneider

Exercise 10 for the lecture

Numerics II

WS 2019/2020

http://numerik.mi.fu-berlin.de/wiki/WS_2019/NumericsII.php

Due: Thursday, January 9th at the tutorial

1. Exercise (4TP)
Consider the DAE

$$Nz'(t) = z(t) + f(t), \qquad z(0) = z_0 \tag{1}$$

with $f \in C^\infty(\mathbb{R}, \mathbb{R}^d)$. Show that if N is nilpotent of degree ν then ν-fold derivation of (1) leads to an ODE without algebraic constraints. What is the differentiation index of (1) in this case ?

2. Exercise (4TP)
Conside the initial value problem for the differential algebraic equation

$$\begin{aligned} y' &= y - z \qquad y(0) = y_0 \\ 0 &= y + z \qquad z(0) = z_0 \end{aligned} \tag{2}$$

a) compute consistent initial conditions y_0, z_0.

b) Solve (2) with these initial conditions by the state space method.

c) Solve (2) with these initial conditions by index reduction.

3. Exercise (3TP)
Show that the differentiation index of a linear differential algebraic system is invariant under equivalence transformations.

4. Exercise (Bonus Points: 2TP+1TP+1PP+2PP)
Consider the Schmitt-trigger initial value problem

$$\begin{pmatrix} C_J & 0 & -C_J & 0 & 0 \\ 0 & C_0 & 0 & -C_0 & 0 \\ -C_J & 0 & 2C_J & -C_J & 0 \\ 0 & -C_0 & -C_J & C_0+C_J & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} u' = - \begin{pmatrix} G_1 u_1 + (1-\alpha) g(u_1 - u_3) \\ G_2 u_2 + G_4(u_2 - u_4) + \alpha g(u_1 - u_3) \\ G_3 u_3 - g(u_1 - u_3) - g(u_4 - u_3) \\ G_4(u_4 - u_2) + (1-\alpha) g(u_4 - u_3) \\ G_5 u_5 + \alpha g(u_4 - u_3) \end{pmatrix} + \begin{pmatrix} G_1 V_{in} \\ G_2 V_{DD} \\ 0 \\ 0 \\ G_5 V_{DD} \end{pmatrix}.$$

a) Transform this problem to the semi-explicit form.

b) Find a consistent initial value for the parameters

$$G = (200, 1600, 100, 3200, 1600), \qquad C_J = 10^{-12}, \qquad C_0 = 40 \cdot 10^{-12},$$
$$g(x) = 10^{-6}\left(\exp\left(\frac{x}{0.026}\right) - 1\right), \qquad \alpha = 0.99, \qquad V_{dd} = 1$$

and the input function

$$V_{in}(t) = 2\sin(2\pi t) + 0.2\sin(20\pi t).$$

c) Solve the problem with the above parameters and initial value numerically on the interval $[0, 2]$ using an appropriate MATLAB method.

d) Solve the problem with the above parameters and initial value numerically on the interval $[0, 2]$ with the state-space method using the Runge-Kutta-4 method.

(*Mind the minus sign in the right hand side !*)

General remarks

You have to do the exercises in groups of up 3 people. Be prepared to demonstrate your solutions to theoretical problems at the given date in the tutorial. Solutions for programming problems have to be submitted via e-mail to xingjian@zedat.fu-berlin.de. with a subject starting by [NumericsII] and denoting all members of the group. Please follow the additional advise for programming exercises on the homepage.