Fachbereich Mathematik & Informatik
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Exercise 11 for the lecture

Numerics II

WS 2019/2020

http://numerik.mi.fu-berlin.de/wiki/WS_2019/NumericsII.php

Due: Thursday, January 16th at the tutorial

1. Exercise (4TP)
Show that Hamiltonian systems are reversible in the following sense. Let

$$y(t) = (p(t), q(t))^T \tag{1}$$

be the unique solution of the Hamiltonian system

$$y'(t) = -J\nabla H(y(t)),\ t \in (0, T], \quad y(0) = y_0, \tag{2}$$

then the backward problem

$$z(t) = -J\nabla H(z(t)),\ t \in (0, T], \quad z(0) = (-p(T), q(T))^T. \tag{3}$$

has the unique solution $z(t) = (-p(T - t), q(T - t))^T$.

2. Exercise (4TP)
Consider the quadratic first integral

$$G(x) = x^T Ax + b^T x + c, \tag{4}$$

of

$$x' = f(x). \tag{5}$$

Prove that $G(x)$ is a first integral if and only if $\nabla G(x) \cdot f(x) = 0, \forall x \in \mathbb{R}^d$.

3. Exercise (4TP)

a) Show that the symplectic Euler method is symplectic.

b) Show that the trapezoidal rule is symplectic if the Hamiltonian is quadratic, i.e. $H(y) = y^T Cy$ holds with a symmetric real matrix C.

c) Show that the trapezoidal rule is not symplectic in general.

4. Exercise (4TP)
Consider the system $q' = p$; $p' = f(q)$.
The explicit one-step method given by

$$\begin{aligned} p_{n+\frac{1}{2}} &= p_n + \frac{\tau}{2} f(q_n) \\ q_{n+1} &= q_n + \tau p_{n+\frac{1}{2}} \\ p_{n+1} &= p_{n+\frac{1}{2}} + \frac{\tau}{2} f(q_{n+1}) \end{aligned}$$

is called Störmer-Verlet method.

Show that the Störmer-Verlet method is symplectic and has second order.

General remarks

You have to do the exercises in groups of up 3 people. Be prepared to demonstrate your solutions to theoretical problems at the given date in the tutorial. Solutions for programming problems have to be submitted via e-mail to xingjian@zedat.fu-berlin.de. with a subject starting by [NumericsII] and denoting all members of the group. Please follow the additional advise for programming exercises on the homepage.