

Exercise 13 for the lecture

NUMERICS II

WS 2019/2020

http://numerik.mi.fu-berlin.de/wiki/WS_2019/NumericsII.php

Due: Thursday, January 30th at the tutorial

1. Exercise (3TP)

Let $A, B \in \mathbb{R}^{n \times n}$ symmetric. Show that

$$\kappa(B^{-1}A) \leq \frac{\mu_1}{\mu_0}$$

holds, if $B \in \mathbb{R}^{n \times n}$ is a preconditioner satisfying

$$\mu_0(Bx, x) \leq (Ax, x) \leq \mu_1(Bx, x) \quad \forall x \in \mathbb{R}^n$$

for some $0 < \mu_0, \mu_1 \in \mathbb{R}$.

2. Exercise (3TP)

The symmetric Gauß-Seidel method for the solution of a linear system with a s.p.d. matrix is obtained by applying one normal Gauß-Seidel step and one Gauß-Seidel step with the components in reversed order alternatingly. Give the iteration matrix and the generated preconditioner of the symmetric Gauß-Seidel method and show that the preconditioner is s.p.d..

3. Exercise (4TP)

Prove the following statements:

- Let $C \in \mathbb{R}^{n \times n}$ be s.p.d.. Then $A \in \mathbb{R}^{n \times n}$ is symmetric with respect to $\langle \cdot, \cdot \rangle_C$ if and only if CA is symmetric.
- Let $A \in \mathbb{R}^{n \times n}$ be diagonalizable, then there is an s.p.d. $C \in \mathbb{R}^{n \times n}$ such that A is symmetric with respect to $\langle \cdot, \cdot \rangle_C$.

4. Exercise (4TP)

Let $A \in \mathbb{R}^{n \times n}$ be s.p.d. Then the Gauß-Seidel method applied to the linear system $Ax = b$ is linearly convergent with a convergence rate $\rho < 1$.

Hint: First show that $G \in \mathbb{R}^{n \times n}$ satisfying $((I - G^*G)x, x) > 0, \forall x \in \mathbb{R}^n$, with some scalar product (\cdot, \cdot) and G^* defined by $(G^*x, y) = (x, Gy), \forall x, y \in \mathbb{R}^n$, has the property $\rho(G) < 1$ and then consider $G = I - (D + L)^{-1}A$.

GENERAL REMARKS

You have to do the exercises in groups of up to 3 people. Be prepared to demonstrate your solutions to theoretical problems at the given date in the tutorial. Solutions for programming problems have to be submitted via e-mail to xingjian@zedat.fu-berlin.de, with a subject starting by [NumericsII] and denoting all members of the group. Please follow the additional advice for programming exercises on the homepage.