

Exercise 14 for the lecture

NUMERICS II

WS 2019/2020

http://numerik.mi.fu-berlin.de/wiki/WS_2019/NumericsII.php

Due: Thursday, February 6th at the tutorial

1. Exercise (4 Bonus TP)

- a) Show that the parallel directional correction method associated with the Euclidean unit vectors e_i is the Jacobi method.
- b) Show that the successive directional correction method associated with the Euclidean unit vectors e_i is the Gauß-Seidel method.

2. Exercise (4 Bonus TP)

Derive from the CG method a method for *non-symmetric* A by application of A^T to $Ax = b$. Which Krylov spaces are spanned by this method? What can you say about the convergence properties?

3. Exercise (6 Bonus PP)

Consider the linear system

$$AU = b \tag{1}$$

with the symmetric positive definite matrix $A \in \mathbb{R}^{n,n}$ and $b \in \mathbb{R}^n$.

- a) Implement the conjugate gradient method and the preconditioned conjugate gradient method as `matlab` functions

```
function [u, uk] = cg(A, b, u0, tol, uexact),
```

and

```
function [u, uk] = pcg(A, b, u0, tol, uexact, pre).
```

`u`, `uk`, `A`, `b`, `u0`, `tol`, and `uexact` denote the last iterate, a vector containing all iterates, the system matrix, the right hand side, the initial iterate, the error tolerance, and the exact solution, respectively. `pre` denotes a function `y = pre(x)` that applies the inverse $y = B^{-1}x$ of some preconditioner B . The iteration should stop if the energy norm $\|\cdot\|_A = \langle A\cdot, \cdot \rangle^{0.5}$ of the error is smaller than the tolerance.

- b) Test your programs with the matrix of the model problem given in the lecture and the right hand side $b = AU$ where U is the pointwise evaluation of $(x_1 - x_1^2)(x_2 - x_2^2)$ for $u_0 = 0$, $\text{tol} = 10^{-8}$ and various choices of n . Use one symmetric Gauß-Seidel step as preconditioner for the pcg-method. Plot the error over the number of iteration steps and compute the average convergence rate for each choice of n . Compare the results with the simple Jacobi and Gauß-Seidel method.

GENERAL REMARKS

You have to do the exercises in groups of up 3 people. Be prepared to demonstrate your solutions to theoretical problems at the given date in the tutorial. Solutions for programming problems have to be submitted via e-mail to xingjian@zedat.fu-berlin.de. with a subject starting by [NumericsII] and denoting all members of the group. Please follow the additional advise for programming exercises on the homepage.