Freie Universität Berlin
Prof. Dr. Ralf Kornhuber

Elliptic partial differential equations
with highly oscillating or random coefficients
WS 2020/2021

3. Exercise sheet, due January 6, 2021

Exercise 1. (4 Points)
Consider the 1D model problem presented in the lecture and prove that the condition number of the stiffness matrix A associated with the nodal basis satisfies

$$\frac{1}{o(1)} \leq K(A) := \frac{\mu_{\max}(A)}{\mu_{\min}(A)} \leq ch^{-2}.$$

Exercise 2. (6 Points)
Consider the 1D model problem presented in the lecture and the splitting of the finite dimensional space S_N into $V_i := \text{span } \lambda_i$, $i = 1, \ldots, N+1$. Prove that this splitting satisfies the conditions $(V1)$ and $(V2)$ introduced in the lecture with constants

$$K_1 \leq c_1 h^{-2} \qquad (1)$$
$$K_2 \leq c_2 \qquad (2)$$

where $c_1, c_2 \in \mathbb{R}$ do not depend on h.

Hint: For the proof of estimate (1) use the following results.

Lemma 1: The discrete L^2-norm

$$|v|_0^2 := \frac{1}{4} h \sum_{i=1}^{N} v(x_i)^2$$

is equivalent to the L^2-norm:

$$c_1 \|v\|^2_{L^2(\Omega)} \leq |v|_0^2 \leq c_2 \|v\|^2_{L^2(\Omega)}$$

where constants c_1, c_2 are independent of h.

Lemma 2: The following estimate holds

$$\|v\|_{H^1(\Omega)} \leq ch^{-1} \|v\|_{L^2(\Omega)}, \quad \forall v \in S_N.$$

Exercise 3. (2 Points)
A projection is orthogonal if and only if it is self-adjoint.