Fachbereich Mathematik & Informatik
Freie Universität Berlin
Prof. Dr. Ralf Kornhuber, Lasse Hinrichsen-Bischoff

Exercise 3 for the lecture

Numerical Mathematics II

WS 2021/2022

http://numerik.mi.fu-berlin.de/wiki/WS_2021/NumericsII.php

Due: Tutorial on November 16, 2021

Problem 1

Discuss the stability of the fixed point $x^* = 0$ of the following ODEs

a)

$$\begin{aligned} x_1' &= -x_1 + x_2^2 \\ x_2' &= -\exp(x_1)x_2 \end{aligned}$$

b)

$$\begin{aligned} x_1' &= \cos(x_1) - \exp(-x_2) \\ x_2' &= x_1 x_2 \end{aligned}$$

Problem 2

Consider the following system of ODEs

$$x'(t) = f(x(t)), \quad f = \begin{bmatrix} x_2 \\ \mu(1 - x_1^2)x_2 - x_1 \end{bmatrix} \tag{1}$$

where μ is a real parameter.

a) Calculate all fixed points of (1).

b) Discuss the (asymptotic) stability of these fixed points depending on parameter μ.

Problem 3
For the ODE from Task 2 solve the ODE numerically for $t \in [0, 10]$ using either one of the methods you implemented on the first problem set or any ODE solver included in Python/Matlab. In the former case, you should use at least $n = 1000$ steps.
Use the following initial value

$$x_0 = \begin{pmatrix} \delta \\ \delta \end{pmatrix}$$

with $\delta = 0.01$.
For each of $\mu \in \{-1, 0, 1\}$, plot the phase diagram and comment on the stability of the fixed point $x^* = 0$ considering your results from the previous task.
Hint: The phase diagram is a plot where the time-axis is only implicit. For each t_k, you want to plot the point (x_{k1}, x_{k2}). It is also a good idea to highlight your initial value and the fixed point in the plot. More in the tutorial.