Fachbereich Mathematik & Informatik
Freie Universität Berlin
Prof. Dr. Ralf Kornhuber, Lasse Hinrichsen-Bischoff

Exercise 4 for the lecture

Numerical Mathematics II

WS 2021/2022

http://numerik.mi.fu-berlin.de/wiki/WS_2021/NumericsII.php

Due: Tutorial on November 23, 2021

Problem 1
Using the notation from Problem 1), show Proposition 1.3.3 from the lecture notes:
If Ψ^τ is consistent with order p, then

$$R(z) = e^z + \mathcal{O}(z^{p+1}) \qquad \text{for } z \to 0.$$

Problem 2
Let $\Psi^\tau : \mathbb{R}^n \to \mathbb{R}^n$ the discrete flow operator of the implicit trapezoidal rule with stepsize τ as applied to the linear system

$$x'(t) = Ax(t).$$

a) Show that Ψ^τ can be written as

$$\Psi^\tau = R(\tau A),$$

with a rational function R of the matrix τA.

b) Derive sufficient conditions on τ for the A-stability of Ψ^τ. Is asymptotic stability inherited from the continuous problem?