Fachbereich Mathematik & Informatik
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Exercise 5 for the lecture

Numerical Mathematics II

WS 2021/2022

http://numerik.mi.fu-berlin.de/wiki/WS_2021/NumericsII.php

Due: 11:59pm on Monday, November 30, 2021

Problem 1

Compute the Butcher scheme for the collocation method with the supporting points of the Simpson rule.

Problem 2

Consider

$$x' = f(x), \tag{1}$$

with $f : \mathbb{R}^d \to \mathbb{R}^d$ and prove the following statements.

a) If f is dissipative, then every fixed point of (1) is stable.

b) If f is strictly dissipative w.r.t. the scalar product $\langle \cdot, \cdot \rangle$ in the sense that there exists a constant $\mu > 0$, such that

$$\langle f(x) - f(\bar{x}), x - \bar{x} \rangle \leq -\mu |x - \bar{x}|^2 \qquad \forall x, \bar{x} \in \mathbb{R}^d \tag{2}$$

holds, then every fixed point of f is asymptotically stable.

Hint: Use Gronwall's lemma to show that

$$|\phi^t x - \phi^t \bar{x}| \leq e^{-2\mu t} |x - \bar{x}| \tag{3}$$

holds for $t \geq 0$ and all $x, \bar{x} \in \mathbb{R}^d$.

Problem 3

Consider the initial value problem

$$x' = Ax, \quad x(0) = x_0 \in \mathbb{R}^d,$$

with $A \in \mathbb{R}^{d,d}$ satisfying $(Av, v) \leq 0 \; \forall v \in \mathbb{R}^d$. Show without using Theorem 1.5.6 that the midpoint rule ($s = 1, b_1 = 1, a_{11} = 1/2$) applied to this initial value problem provides existence and uniqueness of a corresponding approximation for any positive stepsize.