Fachbereich Mathematik & Informatik
Freie Universität Berlin
Prof. Dr. Ralf Kornhuber, Lasse Hinrichsen-Bischoff

Exercise 8 for the lecture

Numerical Mathematics II

WS 2021/2022

http://numerik.mi.fu-berlin.de/wiki/WS_2021/NumericsII.php

Due: 11:59pm on Monday, December 20, 2021

Problem 1
Consider the initial value problem

$$Ex' = Ax, \qquad x(0) = x_0,$$

with $A \in \mathbb{R}^{d\times d}$ and $E \in \mathbb{R}^{d\times d}$ symmetric. Moreover, assume $\ker E = \operatorname{span}\{w\}$, $w \in \mathbb{R}^d$, $w \neq 0$.

a) Derive a necessary condition on x_0 for existence of a solution.

b) Derive a sufficient condition on A for the expliticit solution by the elimination approach.

Problem 2
Derive an exact solution of the initial value problem for the pendulum

$$\varphi'' = -\frac{g}{r}\cos\varphi$$

with initial value

$$x(0) = (0, r_0), \qquad x'(0) = 0.$$

What happens for small perturbation of these initial values?

Problem 3
The pendulum equation is often written as

$$\varphi'' = -\frac{g}{l}\sin\varphi \tag{1}$$

in the literature.

a) In the lecture notes, we derived

$$\varphi'' = -\frac{g}{l}\cos\varphi$$

as the pendulum equation. Explain the difference and motivate the use of equation (1).

b) Do some research on the pendulum equation as stated in (1) and comment once more on the stability of the solution with initial value

$$x(0) = (0, r_0), \qquad x'(0) = 0.$$

Do so *without* taking the linearization into account.