Fachbereich Mathematik & Informatik
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Exercise 9 for the lecture

Numerical Mathematics II

WS 2021/2022

http://numerik.mi.fu-berlin.de/wiki/WS_2021/NumericsII.php

Due: 11:59pm on Monday, January 10, 2022

Problem 1
Consider the DAE

$$Nz'(t) = z(t) + f(t), \qquad z(0) = z_0 \tag{1}$$

with $f \in C^\infty(\mathbb{R}, \mathbb{R}^d)$. Show that if N is nilpotent of degree ν then ν-fold derivation of (1) leads to an ODE without algebraic constraints. What is the differentiation index of (1) in this case ?

Problem 2
Show that the differentiation index of a linear differential algebraic system is invariant under equivalence transformations.

Problem 3
Consider the differential algebraic system

$$\begin{pmatrix} 0 & 0 & 0 \\ c & -c & 0 \\ 0 & 0 & 0 \end{pmatrix} x' = \begin{pmatrix} 1 & 0 & -1 \\ 0 & -\frac{1}{R} & \frac{1}{R} \\ 0 & 0 & 1 \end{pmatrix} x + \begin{pmatrix} -u \\ 0 \\ 0 \end{pmatrix}, \qquad x(0) = x_0 \tag{2}$$

with $c, R, u > 0$.

a) Rewrite this system in normal form, i.e., as

$$\begin{aligned} y'(t) &= Jy(t) + f, & y(0) &= y_0, \\ Nz'(t) &= z(t) + g, & z(0) &= z_0. \end{aligned}$$

Give f, g, J, N, y_0, z_0, and the transformation $T : x \mapsto (y, z)$.

b) How must x_0 be chosen such that there is a unique solution of (2) ?